LAB 2.1: Double Sideband Suppressed carrier

- Amplitude modulation is inherently inefficient because the largest part of the transmitted power is contained in the carrier.
- In suppressed-carrier schemes the carrier is simply not transmitted.
Theoretical Explanation

If \( m(t) \) is a baseband “message” signal and \( \cos(2\pi f_c t) \) is a “carrier” signal at carrier frequency \( f_c \), then we can write the DSB-SC signal \( g(t) \) as

\[
g(t) = A m(t) \cos(2\pi f_c t).
\]

For the special case in which \( m(t) = m_p \cos(2\pi f_m t) \), we can write

\[
g(t) = A m_p \cos(2\pi f_m t) \cos(2\pi f_c t)
\]

\[
= \frac{A m_p}{2} \cos[2\pi (f_c - f_m) t] + \frac{A m_p}{2} \cos[2\pi (f_c + f_m) t].
\]

No Carrier at Frequency \( f_c \)
Figure 1. Double-Sideband Suppressed-Carrier Modulation

Questions:

1. Plot the message signals, carrier signals and modulated signals. 1.5 pts

Hint: Implement equation(1) or (2)
**Expected Results:**

**Figure 1.1:** Block Diagram of Lab2.1

**Simulation parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Denote</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>1</td>
<td>amplifier factor</td>
</tr>
<tr>
<td>$f_m$</td>
<td>1 kHz</td>
<td>frequency of message signal</td>
</tr>
<tr>
<td>$f_c$</td>
<td>20 kHz</td>
<td>frequency of carrier signal</td>
</tr>
<tr>
<td>$T_s$</td>
<td>$0.05 \times 10^{-4}$ s</td>
<td>sampling time</td>
</tr>
</tbody>
</table>
2. Plot the power spectra of message signal, carrier and modulated signal.  

![Figure 1.3: Spectrum of message, carrier and modulated signals](image)

3. Describe the difference in spectra with respect Am modulation carried out in Lab 1.
Lab 2.2 Amplitude demodulation

- Coherent (or synchronous) demodulation
  - In coherent detection, a reference signal of the same frequency and phase as the carrier signal is used.

\[ v(t) = s(t) \cdot c(t) \]

- The carrier has the same frequency and the phase, as of the carrier used in DSBSC modulation.

Message signal: \( m(t) \)
AM modulated signal: \( s(t) \)
Carrier signal: \( c(t) \)
Output of product modulator: \( v(t) \)

\[ s(t) = A_c \cos(2\pi f_c t) m(t) \]
\[ c(t) = A_c \cos(2\pi f_c t + \phi) \]

LO frequency at receiver
Carrier frequency used for modulation
Theoretical analysis

\[ v(t) = A_c \cos(2\pi f_c t) m(t) A_c \cos(2\pi f_c t + \phi) \]

\[ = A_c^2 \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) m(t) \]

\[ = \frac{A_c^2}{2} [\cos(4\pi f_c t + \phi) + \cos \phi] m(t) \]

How to extract message signal?

Low pass filtering: \[ v_0(t) = \frac{A_c^2}{2} \cos \phi m(t) \]

The amplitude is maximum at phase 0, and the signal is 0 at phase shift of 90 degree. Therefore the frequency of modulated carrier and the receiver’s local oscillator should be synchronized.
**Simulation Parameters:**

$F_m = 1 \text{ KHz}, \quad F_c = 20 \text{ KHz}, \quad \text{Sampling time} = 0.05 \times 10^{-4}, \quad \mu = 1$.

Passband edge frequency = 2 KHz, stopband edge frequency = 10 KHz

**Questions:**

1. Evaluate the time and frequency domain equations of the demodulated signals.  
2. Plot the modulated signal and the demodulated signal. (Scale the demodulated signal if necessary).  
3. Plot the spectra of modulated, demodulated signal (before and after low pass filtering).

**Expected Results:**
Expected Results
Lab 2.3 Image Rejection

• What is Image rejection and why is it needed?
  • Receivers have filters designed for certain specific frequency called Intermediate frequency (IF).
  • Not feasible to change the IF frequency to demodulate every signal transmitted at different frequency – Fixed IF frequency with variable LO frequency.
• IF Frequency is the difference between carrier frequency and LO frequency.
• Lower the IF better is the performance.
- **Image**: The signal that is present at IF frequency away from the LO frequency.

- The frequency of local oscillator is chosen which is IF away from the desired signal frequency.

- \( W_{LO} = W_i - W_{IF} \)

- Another signal might be present at the IF frequency away from the local oscillator.

**Solution: Image Rejection!**
- Image Reject Filter: The idea is to remove the signal that is at position $w_{LO} + W_{IF}$.
Theoretical Analysis

- Let the AM signal be

\[ r(t) = A_r \left[ 1 + \mu \frac{m(t)}{m_p} \right] \cos(2\pi f_c t + \theta). \quad (1) \]

Frequency conversion is carried out in hardware by multiplying the received signal by \( \cos(2\pi f_{LO} t) \) and by \( -\sin(2\pi f_{LO} t) \).

The receiver combines the in-phase and quadrature signals to form the complex IF signal given by

\[
\begin{align*}
    r(t) \cos(2\pi f_{LO} t) &= A_r \left[ 1 + \mu \frac{m(t)}{m_p} \right] \cos(2\pi f_c t + \theta) \cos(2\pi f_{LO} t) \\
    &= \frac{1}{2} A_r \left[ 1 + \mu \frac{m(t)}{m_p} \right] \cos[2\pi (f_c - f_{LO}) t + \theta] \\
    &\quad + \frac{1}{2} A_r \left[ 1 + \mu \frac{m(t)}{m_p} \right] \cos[2\pi (f_c + f_{LO}) t + \theta].
\end{align*}
\]

\[ f_{IF} = f_c - f_{LO} \]  

\[ f_{IF} = f_c - f_{LO} \]  

Filter high frequencies

\[ r_I(t) = A_r \left[ 1 + \mu \frac{m(t)}{m_p} \right] \cos(2\pi f_{IF} t + \theta). \quad (3) \]
• Quadrature component:

The receiver also forms a second signal,

\[ -r(t)\sin(2\pi f_{LO}t) = -A_r \left[ 1 + \mu \frac{m(t)}{m_p} \right] \cos(2\pi f_c t + \theta)\sin(2\pi f_{LO}t) \]

\[ = \frac{1}{2} A_r \left[ 1 + \mu \frac{m(t)}{m_p} \right] \sin\left[ 2\pi (f_c - f_{LO}) t + \theta \right] \]

\[ - \frac{1}{2} A_r \left[ 1 + \mu \frac{m(t)}{m_p} \right] \sin\left[ 2\pi (f_c + f_{LO}) t + \theta \right]. \]

Again, the high-frequency term is removed, and the receiver provides the “quadrature” signal

\[ r_Q(t) = A_r \left[ 1 + \mu \frac{m(t)}{m_p} \right] \sin(2\pi f_{IF} t + \theta). \]
Complex IF signal:

\[
\tilde{r}(t) = r_I(t) + j r_Q(t) = A_r \left[ 1 + \mu \frac{m(t)}{m_p} \right] \left[ \cos(2\pi f_{IF} t + \theta) + j \sin(2\pi f_{IF} t + \theta) \right]
\]

\[
= A_r \left[ 1 + \mu \frac{m(t)}{m_p} \right] e^{j(2\pi f_{IF} t + \theta)}.
\]

In the frequency domain, the spectrum \( R(f) \) of the received signal \( r(t) \) given in Eq. (1) is shown in Figure 1.

![Figure 1. Spectrum of Received AM Signal](image)
Since $r(t)$ is a real-valued signal, its spectrum contains both positive and negative-frequency components. After frequency conversion, the complex IF signal $\tilde{r}(t)$ has the spectrum $\tilde{R}(f)$ shown in Figure 2.

![Figure 2. Spectrum of Complex IF Signal](image)

Notice that $\tilde{r}(t)$ contains only positive frequency components.
We passed the complex IF signal through a bandpass filter. Figure 3 shows the frequency response of the bandpass filter. It is intended that signals at carrier frequencies other than $f_{IF}$ will be rejected by the filter.

Figure 3. Frequency Response of Intermediate-Frequency Filter
Image Signal

Suppose that there is a second signal received along with the signal of Eq. (1), and that this signal is given by

\[ r_2(t) = A_{r_2} \left[ 1 + \mu_2 \frac{m_2(t)}{m_{2p}} \right] \cos(2\pi f_{IM}t + \theta_2), \quad (7) \]

where the carrier frequency \( f_{IM} \) happens to be given by

\[ f_{IM} = f_{LO} - f_{IF}. \quad (8) \]

If we carry out the analysis of Eqs. (7) and (8), we find that this second signal produces the complex IF signal \( \tilde{r}_2(t) \) given by

\[ \tilde{r}(t) = A_{r_2} \left[ 1 + \mu_2 \frac{m_2(t)}{m_{2p}} \right] e^{j(-2\pi f_{IF}t + \theta_2)}. \quad (9) \]
The spectrum $\tilde{R}_2(f)$ of this signal is shown in Figure 4.

Figure 4. Spectrum of the Image Signal

The signals $r(t)$ and $r_2(t)$ are said to be "images" of one another. A glance at the frequency response shown in Fig. 3 shows that both $r(t)$ and $r_2(t)$ will pass through the IF filter, and the two signals will interfere with each other in the demodulator that follows the IF filter. The relationship between the frequencies of the two image signals is worth noting. One signal, $r(t)$, is at a carrier frequency of $f_c = f_{LO} + f_{IF}$, while the other signal, $r_2(t)$, is at a carrier frequency of $f_{IM} = f_{LO} - f_{IF}$. These carrier frequencies are symmetrically arranged about the receiver's frequency $f_{LO}$, the way a physical object and its image are symmetrically distant from the surface of a mirror.
1. Plot the spectra of $r(t)$, $\tilde{r}(t)$, when $m(t) = \cos(2\pi 50t)$ and for $m(t) = \cos(2\pi 500t) + j\sin(2\pi 500t)$.

**Simulation Parameters:** $F_c = 20$ KHz, $F_{lo} = 20.5$ KHz, $F_{lf} = 500$ Hz

**Expected Results:**

Spectrum of real and complex signals (a) $r(t)$; (b) $\tilde{r}(t)$; with $m_1(t) = \cos(2\pi f_m t)$, $f_m = 50$ Hz

![Graphs showing spectra](image)
Spectrum of real and complex signals (a) $r(t)^2$; (b) $\tilde{r}(t)^2$; with $m_2(t) = \cos(2\pi f_m t) + j\sin(2\pi f_m t)$, $f_m = 50\, \text{Hz}$
NOTE: The deadline for Lab2 simulink model submission is 10\textsuperscript{th} July

Thank you!